Model Theoretic Semantics

Pseudo Haskell:
(a) set notation with curly braces
(b) magical applicative brackets
(c) logical language expressions in this font

Vocabulary

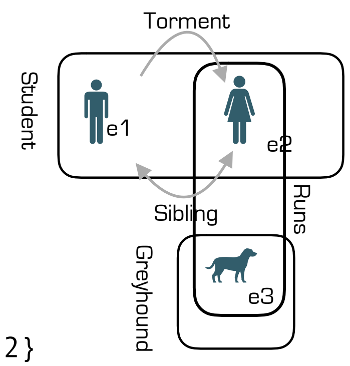
```
data Vocab =
   V0 Thing
 | V1 Rel1
 | V2 Rel2

data Thing =
   T Bart
 | T Lisa
 | T SantaLH

data Rel1 =
   U Student
 | U Greyhound
 | U Run

data Rel2 =
   B Sibling
 | B Torment
```

Language

```
newtype Variable =
   Variable String

data Term =
   Const Thing
 | Var Variable

data Formula =
   UnaryRel Rel1 Term
 | BinaryRel Rel2 Term Term
 | Not Formula
 | And Formula Formula
 | Or Formula Formula
 | Implies Formula Formula
 | Exists Variable Formula
 | ForAll Variable Formula
```

Model

```
data Entity = E1 | E2 | E3 |..
  deriving (Bounded)

type Model = Vocab → {Entity}
model9871 :: Model
model9871 (V0 Bart)          = {E1}
model9871 (V0 Lisa)          = {E2}
model9871 (V0 SantaLH)       = {E3}
model9871 (V1 (U Student))   = {E1, E2}
model9871 (V1 (U Greyhound)) = {E3}
model9871 (V1 (U Runs))      = {E2, E3}
model9871 (V2 (B Sibling))   = {(E1, E2), (E2, E1)}
model9871 (V2 (B Torment))   = {(E1, E2)}
```



Assignment

```
type Assignment = Variable → Maybe Entity

assign3193 :: Assignment
assign3193 (Variable "x") = Just E1
assign3193 (Variable "y") = Just E2
assign3193 _ = Nothing
```

```
data Formula2
   Lambda Variable Formula
 | Apply  Variable Formula Formula2
 | NForm  Formula
```

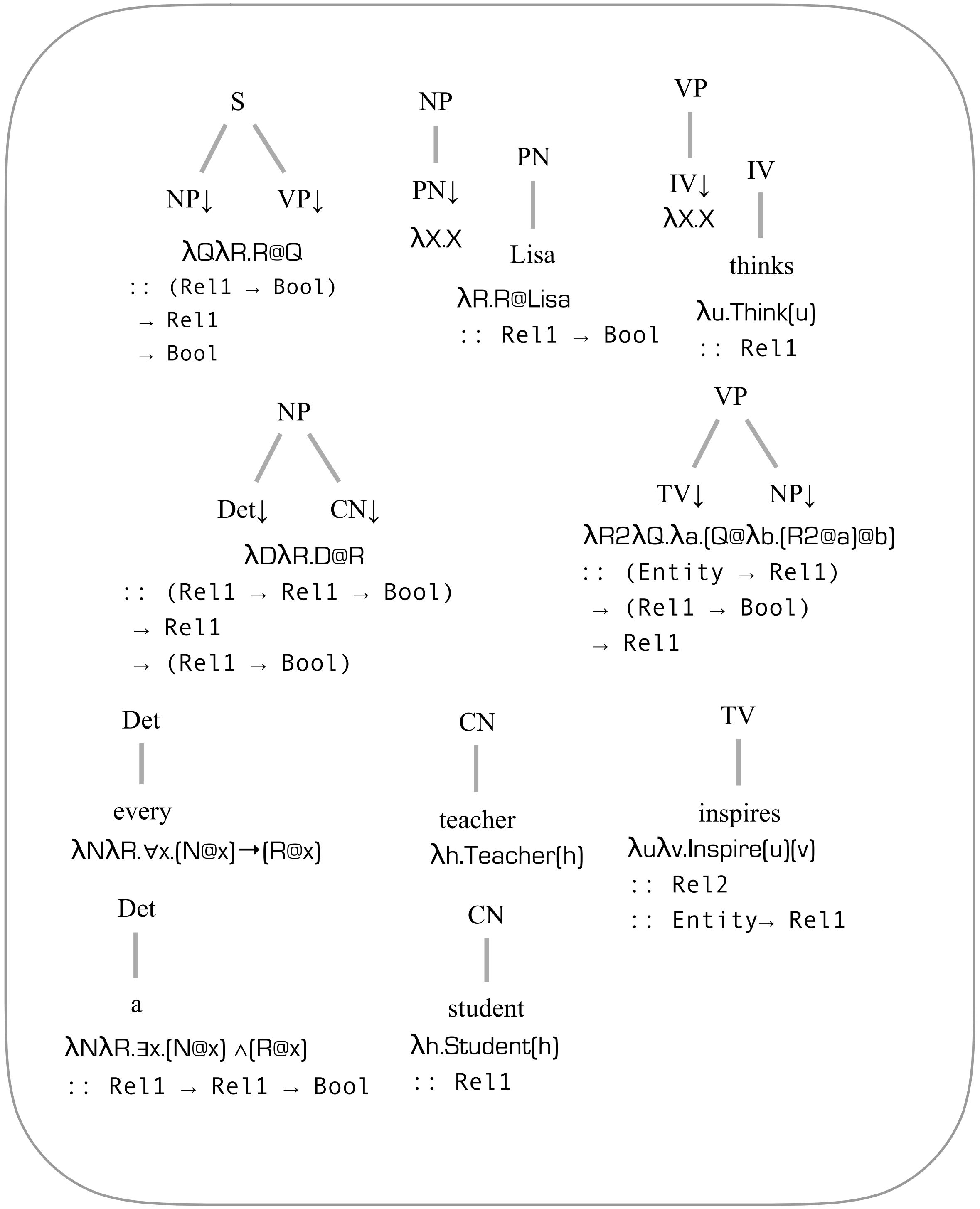
We will see lambdas in Part 3 of the talk
Here in the handout, I do not bother distinguishing between lambda variables and logical variables

Interpretation

```
class Interp res v where
  interp :: Model → Assignment → v → Maybe res

instance Interp Entity Variable where
  interp mdl asg v = asg v
instance Interp Entity Vocab where
  interp mdl asg v = Just (mdl v)
instance Interp Entity Thing where
  interp mdl asg t = interp mdl asg (V0 t)
instance Interp Entity Term where
  interp mdl asg (Constant c) = interp mdl asg c
  interp mdl asg (Var v)      = intern mdl v

instance Interp Bool Formula where
  interp mdl asg f = satisfies mdl asg f

satisfies :: Model → Assignment → Formula → Maybe Bool
satisfies mdl asg f = case f of
   UnaryRel  r t     -> 〈 int t ∈ int r 〉
   BinaryRel r t1 t2 -> 〈(int t1, int t2) ∈ int r〉
   Not f        -> 〈 not (sat f) 〉
   And f1 f2    -> 〈 sat f1 && sat f2 〉
   Or f1 f2     -> 〈 sat f1 || sat f2 〉
   Implies p q  -> 〈 not (sat f1) || sat f2 〉
   Exists x f   -> 〈 any (\a -> satisfies mdl a f)
                          (variants asg v) 〉
   ForAll x f   -> 〈 all (\a -> satisfies mdl a f)
                          (variants asg v) 〉
  where
   int = interp mdl asg
   sat = satisfies mdl asg f

variants :: Assignment → Variable → [Assignment]
variants asg v0 =
  [ \v -> if v == v0 then Just e else asg v
  | e <- [minBound..maxBound]] -- for all entities
```

Natural Language Semantics

Something I don't say explicitly in the talk is that we think here of our model as having two types of objects in its domain: Entities and Truth Conditions
Montague's Intensional Logic *has more stuff: Senses, Points in Time, Worlds*




```
type Rel1 = Entity → Bool
type Rel2 = Entity → Entity → Bool
```

⟦Lisa thinks⟧	`:: Bool`
⟦Lisa inspires a student⟧	`:: Bool`
⟦Lisa⟧	`:: Rel1 → Bool`
⟦a student⟧	`:: Rel1 → Bool`
⟦thinks⟧	`:: Rel1`
⟦inspires a student⟧	`:: Rel1`
⟦inspires⟧	`:: Entity → Rel1`

Example derivation